A New Analysis and Design Method for Fuzzy Logic Controllers Used in Power Converters

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Abstract – This paper proposes a novel design procedure of fuzzy logic controller (FLC) for DC-to-DC converters that integrates linear control techniques with fuzzy logic. The design procedure allows the small signal model of power converters and linear control design techniques to be used in the initial stages of FLC design. This simplifies the small signal design and the stability assessment of the FLC. By exploiting the fuzzy logic structure of the controller, heuristic knowledge is incorporated into the design, resulting in a non-linear controller with improved large signal performance over linear PI controllers.

I. INTRODUCTION

Fuzzy logic control (FLC) has been applied successfully to a wide variety of engineering problems, including DC-to-DC converters [1]-[3]. It is an attractive control method because its structure resembles the way human intuitively approaches a control problem. This makes it easy for the designer to incorporate heuristic knowledge of a system into the controller. Fuzzy control is of great value for problems where linearity, and/or imprecision. Power converters fall into this category since they have a time-varying structure and contain elements that are non-linear and have parasitic components.

Despite its advantages, there are some problems with fuzzy logic control. Design of fuzzy controllers is usually accomplished by trial and error [1]-[3]. Stability analysis can be difficult and usually involves the use of non-linear control techniques [4],[5]. With fuzzy logic control, it can be harder to assess the performance of the controller than with linear control methods where metrics such as bandwidth can be used. Using linear control techniques, it can be known how changing control parameters will affect performance (bandwidth) and stability (phase and gain margins), but there is no analogous approach for fuzzy logic control.

In recent years, research shows that fuzzy control can be directly related to the linear control [6]-[8]. Using this knowledge, a new design methodology of fuzzy logic controllers for DC-DC converters, which integrates the advantages of linear control techniques and fuzzy logic control, is developed in this paper. In this method, linear models and linear control techniques are used in the initial design of the fuzzy controller. This initial controller has exactly the same response as a linear PI controller. As a result, its stability and performance can be assessed using simple linear control techniques and well known small signal model of the converter. The amount of trial and error in the initial design of the fuzzy logic controller is reduced.

Furthermore, heuristic knowledge is incorporated by capitalizing on the fuzzy logic implementation of the controller. This can give an improved non-linear controller that outperforms its linear counterpart in the initial design. The large signal performance of the controller can then be improved without compromising the stability or performance of the controller for small signals.

In this paper, the proposed design methodology is developed for PI controllers since these are simple two term controllers (compared to PID control which requires three terms) and give zero steady state error (compared to PD controllers which have steady state error). Simulation and experimental results show that the proposed FLC has the same small signal performance as linear PI controller. In addition, the designed FLC has much better large signal dynamic performance than linear PI controller.

II. RELATIONSHIP BETWEEN THE PROPOSED FUZZY LOGIC CONTROL AND LINEAR PI CONTROL

This section will discuss the relationship between a PI-like fuzzy logic controller and a digital linear PI controller. The block diagram for a digital implementation of PI-like FLC is given in Fig. 1. The error e(k) is sampled with a sampling period Ts. The inputs to the FLC are the error e(k) and the change of error, Δe(k), which is computed as:

\[ Δe(k) = e(k) - e(k - 1) \]  \hspace{1cm} (1)

The output of the fuzzy logic controller is an incremental change in the control signal, Δu(k). The control signal u(k) is obtained as

\[ u(k) = Δu(k) + u(k - 1) \]  \hspace{1cm} (2)

A Sugeno-type fuzzy controller with the following constraints is used as the proposed PI-like FLC:
1. Input membership functions are triangular except for the left most and right most membership function (shown in Fig. 2).
2. For any value of each controller input, at most only two input membership functions are active and the sum of the membership for all active fuzzy sets is one.
3. The output membership functions are singletons (crisp values).

**Fig. 1 Digital PI-like fuzzy logic controller**

4. The ‘AND’ operation in the rule antecedent is performed by multiplication.
5. Active rules are combined by the “or” operation, which is accomplished by addition.
In Fig. 2, the variable $x$ is a generic variable; in the case of the PI-like FLC given in Fig. 1, $x$ could represent the error $e$ or the change of error $\Delta e$. It should be mentioned that the input membership functions are not necessarily evenly distributed as illustrated in Fig. 2.

Based on above constraints of the proposed fuzzy logic controller, it can be known that if $x_k < x < x_{k+1}$, the degree of membership of the input can be calculated as:

$$\mu_{P_{x_{k+1}}}(x) = \frac{x - x_k}{x_{k+1} - x_k}$$

(3)

$$\mu_{P_x}(x) = 1 - \frac{x - x_k}{x_{k+1} - x_k} = 1 - \mu_{P_{x_{k+1}}}(x)$$

(4)

where $\mu_{P_{x_{k+1}}}$ is the degree of membership of $x$ to $P_{x_{k+1}}$, $\mu_{P_x}$ is the degree of membership of $x$ to $P_x$. $x_k$ is the point where $x$ has full membership to $P_k$, and $x_{k+1}$ is the point where $x$ has full membership to $P_{k+1}$. There are two exceptions where (3) and (4) cannot be used to calculate membership. If $x < x_k$, then $\mu_{P_{x_{k+1}}}(x) = 1$ and degree of membership is zero for all other fuzzy sets. If $x > x_n$, then $\mu_{P_{x_{k+1}}}(x) = 1$ and degree of membership is zero for all other fuzzy sets.

In the proposed two-input single-output PI-like fuzzy logic controller, rules are of the form: “If $e$ is $A_k$ and $\Delta e$ is $B_k$ then $\Delta u = \Delta u_{A_k B_k}$”, where $A_k$ and $B_k$ are fuzzy sets on the error and change of error inputs respectively. Because for each input at most two fuzzy sets have non-zero membership, for an input $(e, \Delta e)$ to the fuzzy controller where $e$ belongs to $A_k$ and $A_{k+1}$, $\Delta e$ belongs to $B_k$ and $B_{k+1}$, at most four rules are activated. These rules are:

1. If $e$ is $A_k$ and $\Delta e$ is $B_k$, then $\Delta u = \Delta u_{A_k B_k}$;
2. If $e$ is $A_{k+1}$ and $\Delta e$ is $B_k$, then $\Delta u = \Delta u_{A_{k+1} B_k}$;
3. If $e$ is $A_k$ and $\Delta e$ is $B_{k+1}$, then $\Delta u = \Delta u_{A_k B_{k+1}}$;
4. If $e$ is $A_{k+1}$ and $\Delta e$ is $B_{k+1}$, then $\Delta u = \Delta u_{A_{k+1} B_{k+1}}$.

The output of the proposed FLC is the weighted sum of all the activated rules and is given by

$$\Delta u = (\mu_{A_k} \cdot \mu_{B_k}) \cdot \Delta u_{A_k B_k} + (\mu_{A_{k+1}} \cdot \mu_{B_k}) \cdot \Delta u_{A_{k+1} B_k} + (\mu_{A_k} \cdot \mu_{B_{k+1}}) \cdot \Delta u_{A_k B_{k+1}} + (\mu_{A_{k+1}} \cdot \mu_{B_{k+1}}) \cdot \Delta u_{A_{k+1} B_{k+1}}$$

(5)

It is noted that since the output membership functions are just singletons (crisp values), defuzzification is simplified.

With the constraints described above, the proposed fuzzy logic controller has two special properties that allow it to be assigned a known piecewise linear control surface:

1. **Property 1**: Each rule defines the output of the controller (a point on the control surface) when the control inputs have full membership to the fuzzy sets in the rule antecedent. The controller output for this case is equal to the consequent of the rule.
2. **Property 2**: If the consequent of each of the four active rules lie in a plane on the control surface, then all points calculated by the controller using these rules will lie in that plane.

**Proof of Property 1**: Consider the rule “If $e$ is $A_k$ and $\Delta e$ is $B_k$, then $\Delta u = \Delta u_{A_k B_k}$”. At $e = e_k$, $e$ has full membership to $A_k$, and at $\Delta e = \Delta e_k$, $\Delta e$ has full membership to $B_k$. It can be seen from (5) that if the input is $(e_k, \Delta e_k), \mu_{A_k} = 1, \mu_{B_k} = 1$ and $\mu = 0$ for all other fuzzy sets, the output is just $\Delta u = \Delta u_{A_k B_k}$. In terms of a control surface, this means that the point $(e_k, \Delta e_k, \Delta u = \Delta u_{A_k B_k})$ is a point on the surface. This is also true for all the other rules. This property arises from the restrictions placed on the membership functions that implied when the degree of membership is equal to 1 for one fuzzy set, it is zero for all others.

**Proof of Property 2**: Consider a plane defined by three points in a three-dimensional space, which is determined by the variables $e$, $\Delta e$ and $\Delta u$. The points are called $M(e_1, \Delta e_1, \Delta u_1), N(e_2, \Delta e_1, \Delta u_2), P(e_1, \Delta e_2, \Delta u_3)$.

As illustrated in Fig. 3, using linear algebra, this plane can be described by the following equation:

$$\Delta u = \Delta u_1 + \frac{e - e_1}{e_2 - e_1} (\Delta u_2 - \Delta u_1) + \frac{\Delta e - \Delta e_1}{\Delta e_2 - \Delta e_1} (\Delta u_3 - \Delta u_1)$$

(6)

We now try to find a fourth point $Q$, which also lies in the plane described by (6). It is assumed that $Q$ lies at $e = e_2$ and $\Delta e = \Delta e_2$ and has a $\Delta u$ coordinate $\Delta u_4$. To make $Q$ lie in the plane, by using (6), it can be found that

$$\Delta u_4 = \Delta u_2 + \Delta u_3 - \Delta u_1$$

(7)

![Fig. 2 Input membership functions of the proposed fuzzy logic controller](image-url)

![Fig. 3 Points in a three dimensional space](image-url)
For the proposed Sugeno-type fuzzy logic controller, its output can be calculated by (3), (4) and (5). Assuming that for each input $x$, $x_l < x < x_u$, so that (3) and (4) can be used. Without loss of generality, it can be assumed that $e_l < e < e_u$ and $\Delta e_l < \Delta e < \Delta e_u$. In terms of notation in (3), (4) and (5), this means $e_l = e_1, e_u = e_2, \Delta e_l = \Delta e_1, \Delta e_u = \Delta e_2, A_k = A_1, A_{k+1} = A_2, B_k = B_1, B_{k+1} = B_2$. Substituting this notation and (3), (4) into (5), it can be obtained

$$\Delta u = \Delta u_{1,n} + \mu_{\Delta e_1}(-\Delta u_{1,n} + \Delta u_{2,n}) + \mu_{\Delta e_2}(-\Delta u_{1,n} + \Delta u_{2,n})$$

(8)

where $\mu_{\Delta e_1} = \frac{e - e_1}{e_2 - e_1}$ and $\mu_{\Delta e_2} = \frac{\Delta e - \Delta e_1}{\Delta e_2 - \Delta e_1}$ can be derived from (3) respectively.

Assuming that the points $M, N, P, Q$ are defined by (9), (3), (4) into (5), it can be obtained

$$\mu_{\Delta e_1} = \mu_{\Delta e_2} = \frac{\Delta e - \Delta e_1}{\Delta e_2 - \Delta e_1}$$

(9)

Since the last term of (9) is zero, it is simplified to

$$\Delta u = \Delta u_{1,n} + \mu_{\Delta e_1}(-\Delta u_{1,n} + \Delta u_{2,n}) + \mu_{\Delta e_2}(-\Delta u_{1,n} + \Delta u_{2,n})$$

(10)

It can be seen that equation (10) is the same as equation (6), which describes the plane found in the beginning of the proof. This means the control law of (10) describes a plane (for the active rules) if the four points described by the active rules lie in a plane. This is the second special property.

The proposed fuzzy controller will now be compared to a digital PI controller to obtain a relationship between these two controllers. The transfer function for a continuous PI controller, $C(s)$, with parameters $a$ and $G$ is given by

$$C(s) = \frac{U(s)}{E(s)} = \frac{a \cdot s + 1}{s}$$

(11)

The bilinear transformation, given in (12), can preserve the phase and gain properties of the controller below approximately 1/10 of the sampling frequency [9]. Therefore, it is used to obtain a discrete equivalent of continuous PI controller as shown in (13),

$$s = \frac{2}{T_s} \left( \frac{1 - z^{-1}}{1 + z^{-1}} \right)$$

$$C(z) = \frac{U(z)}{E(z)} = \frac{m \cdot z + n}{z - 1}$$

where the parameters $m$ and $n$ are given by

$$m = G \left( a + \frac{T_s}{2} \right)$$

$$n = G \left( \frac{T_s}{2} - a \right)$$

(15)

In addition, the transfer function of (13) can be expressed as a difference equation

$$u(k) = (m + n)e(k) - n(e(k) - e(k - 1)) + u(k - 1)$$

(16)

A block diagram representation of this difference equation is illustrated in Fig. 4. The difference between Fig. 1 and Fig. 4 is that the dashed box of Fig. 4 is replaced by a two-input single-output FLC in Fig. 1. It is noted that this dashed box in Fig. 4 has the relationship:

$$\Delta u(k) = (m + n)e(k) - n \cdot \Delta e(k)$$

(17)

It is well known that the input-output relationship of linear PI controller (shown in (17)) represents a plane. In addition, it is proven in this section that the proposed fuzzy controller also generate a planar control surface (property 2). By using property 1 of the proposed FLC, we can assign the rules of the fuzzy controller to make FLC have points that lie in the same plane as equation (17). This can be achieved by the following method:

For the rule “If $e$ is $A_k$ and $\Delta e$ is $B_k$, then $\Delta u = \Delta u_{1,n}$”, the value for $\Delta u_{1,n}$ is initialized as

$$\Delta u_{1,n} = e_1 \cdot (m + n) + \Delta e_1 \cdot (-n)$$

(18)

Thus, a fuzzy logic controller can be made to give the same input-output relationship as the digital PI controller (17). The significance of the proposed design method is that, linear control techniques and small signal model can be used to design a fuzzy logic controller. This reduces the amount of trial and error, and gives fuzzy logic controller with predictable small signal stability and performance.

III. DESIGN METHODOLOGY TO IMPROVE THE CONTROLLER PERFORMANCE USING FUZZY LOGIC

Fuzzy logic control is also a non-linear control technique, which enables it to give different control action for large and small signals; this is not possible with linear PI control. It therefore seems reasonable to start with a linear controller designed using the small-signal model and make it non-linear using heuristic knowledge of the system. In order to further improve the performance of the proposed FLC, the following heuristic knowledge rules of the Buck converter can be used:

1. If the error is far from zero, then the change in duty cycle should be large.
2. If the error is near zero, then the change in duty cycle should be small.

![Fig. 4 Block diagram representation of equation (13)](image-url)
3. If the error is near zero, but the change of error is large, the duty cycle should be changed to prevent overshoot.
4. If the change of error is in the direction such that the output is approaching the zero error, but the error is not close to zero, then the change in duty cycle need not be as large as if the change of error were in the opposite direction.

A flowchart of the design procedure is shown in Fig. 5. This procedure consists of three basic steps. In the first step, a conventional linear digital controller is designed. The second step transfers this controller to a fuzzy logic implementation. In the third step, this fuzzy logic implementation is exploited to incorporate heuristic knowledge resulting in a controller with the improved large signal performance.

Step 1 of Fig. 5 shows the procedure of designing a digital controller. The control system of power supplies is usually implemented using analog electronics. As a result, many designers are accustomed to designing the controllers in the continuous time domain. For this reason, the design by emulation technique is adopted. At first, PI compensator is designed in the continuous time domain. The gain and zero of a continuous PI controller of equation (11) are chosen to provide the desired response. The bilinear transformation (12) is then used to find a digital equivalent, resulting in the digital PI compensator of (13).

Once a digital controller has been designed, it can be transferred to the fuzzy controller as described in section II. This is the step 2 of the design procedure. The first property of fuzzy logic controller described in section II is used. It should be noted that, the proof of the second property in section II makes the assumption that for each input $x$ to the controller, $x_1 < x < x_0$. This means that the values of $x_1$ and $x_0$ need to be chosen appropriately, so that property 2 will hold over the range of inputs expected by the controller. The number and distribution of the membership functions are chosen based on trial and error. More membership functions give more controller parameters, and thus more freedom to shape the control surface. Putting membership functions closer together means that there are more parameters to describe that region of control surface, and therefore more freedom to shape that region.

In step 3, heuristic knowledge of the system and trial and error are used to improve the performance of the controller. This knowledge can be included by altering the input membership functions or the consequents of the rules (output singletons). In addition, the membership functions and rules are unaltered in a small region near the steady state operating point of the controller (zero error and zero change of error). Then, the input-output relationship of the controller will remain unchanged in this region. As a result, the performance of the controller for inputs constrained in this region will be the same as for the initial controller and can be predicted using the small signal models and linear control techniques.

The proposed design procedure is applied to a synchronous Buck converter. The parameters of this converter are $V_{in}=5V$, $V_o=2.5V$, $L=1\mu H$, $C=220\mu F$, $R_s=2m\Omega$, $EJR=1m\Omega$. $R_s$ is the winding resistor of inductor and ESR the equivalent series resistor of the output capacitor. The switching frequency is 400kHz.

A continuous PI controller is designed with a transfer function $C(s)=\frac{2000(1+0.000125s)}{s}$ to achieve the highest bandwidth and $50^\circ$ phase margin. It is shown that in this PI controller, $G=2000$ and $\alpha=0.000125$.

In the proposed PI-like fuzzy logic controller, nine membership functions $A_1$ through $A_9$ were designated for the error input $e$. For the change of error input $\Delta e$, nine input membership functions were designated $B_1$ through $B_9$. It is noted that $e_1$ is the point where $\mu_i=1$ and $\Delta e_i$ is the point where $\mu_{B_i}=1$. The values of $e_1$ through $e_9$ and $\Delta e_1$ through $\Delta e_9$ are given in Table 1 and Table 2 respectively. The membership functions are illustrated in Fig. 6-Fig. 7.

The rule table for the proposed fuzzy logic controller is given in Table 3. Each entry gives the change in duty cycle $\Delta u$, when membership is full to both the corresponding fuzzy sets in the rule antecedent. For instance, the entry in row 2, column 3 gives the normalized change in duty cycle if membership is full to the membership function $A_2$ on the error input and the membership function $B_3$ on the change of error input.

The rules were initialized as discussed in section II. For example, for the rule “If $e$ is $A_2$ and $\Delta e$ is $B_3$ then $\Delta u = u_{A_2B_3}$”, the value for $u_{A_2B_3}$ is initialized to

$$u_{A_2B_3} = e_2 \cdot (m+n) + \Delta e_3 \cdot (-n)$$

$$= -1 \cdot (0.2025 + (-0.1975)) + (-0.016) \cdot (-(-0.1975))$$

$$= -0.081$$

where $m$ and $n$ can be obtained from (14) and (15).
Membership functions were obtained. That is the definitions previously and trial and error, new definitions of the membership functions, but this can be adjusted to 1, increasing the gain at this point. Using the heuristic knowledge mentioned through dynamic response than the original digital PI controller. It is noted that the control surface near the origin remained the same for small signals and can be predicted by the small signal model.

A comparison of the experimental results of the digital PI controller and the proposed fuzzy logic controller was done in terms of the large signal response. Fig. 10 shows that the proposed fuzzy logic controller has fast dynamic response than digital PI controller under large signal reference voltage change (from 2.5V to 3.0V). Fig. 11 shows that under large signal input voltage change, the overshoot of the proposed FLC is reduced to 65% of that using PI controller. The recovery time is reduced by 50%. Under large signal load change, the undershoot of the proposed FLC is reduced to 80% of that using PI controller (shown in Fig. 12). The recovery time is reduced by 30%. Therefore, it can be observed from these experimental results that the improved fuzzy logic controller has better large signal dynamic response than the original digital PI controller.

### IV. SIMULATION AND EXPERIMENTAL RESULTS

In order to compare the small signal response of digital PI controller and the proposed fuzzy logic controller, simulation was done in frequency domain. From Fig. 8, it can be seen that two controllers have the same small signal frequency response. In addition, experiment was done to impose a small step change of 16mV (from 2.5V to 2.516V) in the reference voltage. Fig. 9 shows that the behaviour of the proposed fuzzy controller and the original digital PI controller is the same for small signals and can be predicted by the small signal model.

A comparison of the experimental results of the digital PI controller and the proposed fuzzy logic controller was done in terms of the large signal response. Fig. 10 shows that the proposed fuzzy logic controller has fast dynamic response than digital PI controller under large signal reference voltage change (from 2.5V to 3.0V). Fig. 11 shows that under large signal input voltage change, the overshoot of the proposed FLC is reduced to 65% of that using PI controller. The recovery time is reduced by 50%. Under large signal load change, the undershoot of the proposed FLC is reduced to 80% of that using PI controller (shown in Fig. 12). The recovery time is reduced by 30%. Therefore, it can be observed from these experimental results that the improved fuzzy logic controller has better large signal dynamic response than the original digital PI controller.

Rules 1 and 2 proposed in the beginning of this section state that the gain should be increased further from the zero error point. This knowledge was included into the design by changing the definitions of the input membership functions. Trial and error was used to pick appropriate values for input memberships. For example, a very large error was considered to be 6 according to original definitions of the membership functions, but this can be adjusted to 1, increasing the gain at this point. Using the heuristic knowledge mentioned previously and trial and error, new definitions of the membership functions were obtained. That is the definitions of $A_1$, $A_2$, $A_3$, $A_4$, and $A_9$ on the error input and $B_1$, $B_2$, $B_3$, $B_7$, $B_8$, and $B_9$ on the change of error input. The new values are given in Table 4 and Table 5.

It is noted that the control surface near the origin remained the same as for the PI controller case, as the membership functions (and rules) were unchanged between −0.16 and 0.16 on each input. Therefore, for very small changes, the response will be the same as the linear PI controller, but for large signal changes, the response will be different.
V. CONCLUSIONS

This paper presented a new design procedure of fuzzy logic controllers for DC-to-DC converters. The proposed technique allows the small signal model of the converter and linear control techniques to be applied in the initial stages of fuzzy controller design. This method makes assessing the small signal performance and stability of the fuzzy controller become easier. The proposed FLC using linear techniques serves as a known starting point from which improved performance can be achieved by applying heuristic knowledge to obtain a non-linear controller. As a result, the large signal dynamic performance is improved. At the same time, the performance and stability of the improved non-linear controller can still be assessed using linear control techniques for small signals if the control surface remains linear in the region in which these small signals fall. A design example was presented with simulation and experimental results to illustrate and verify this procedure.

REFERENCES